ADA UNIVERSITY

SCHOOL OF EDUCATION

MASTER OF ARTS IN EDUCATIONAL MANAGEMENT

MASTER THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS IN EDUCATIONAL MANAGEMENT

Title: UNDERSTANDING LOGARITHM: WHAT ARE THE DIFFICULTIES THAT STUDENTS HAVE?

Students' names: Masima Rahimova

Contact e-mail: mrahimova@ada.edu.az

Date: 10.04.2024

STATEMENT OF AUTHENTICITY

I have read ADA's policy on plagiarism and certify that, to the best of my knowledge, the content of this paper, entitled "Understanding Logarithm: What are the Difficulties That Students Have?", is all my own work and does not contain any unacknowledged work.

Signed: ______________________

Date: ______________

School of Education

Public Policy and Strategy: Master's Thesis

Approval Form

Student Name/Surname: Masima Rahimova

Student ID number: 000012963

Program Name: MA in Educational Management

Academic Track Selection:

Research Track ☐ Professional Track **V**

Comments:

The Course Instructor:

The Supervisor:

SE Curator of the Graduate Programs:

Dean of SE:

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Abstract

Calculus, linear algebra, and differential equations are more complex areas of mathematics, and students may have difficulty moving forward in these courses due to their difficulties with logarithms. In order to help students succeed in mathematics and related fields in the long run, it is crucial to identify and resolve these obstacles in higher education. The aim of this thesis is to investigate the challenges high school students in Baku, Azerbaijan, face in understanding logarithms, crucial for success in advanced mathematics and related disciplines. It examines the relationship between students' performance on logarithmic tasks and teachers' perceptions of their understanding and abilities. The research reveals that students' difficulties primarily stem from a lack of foundational mathematical skills and a deep conceptual understanding of logarithms. The study employs a mixed-methods approach, combining thematic analysis of teacher interviews with descriptive statistical analysis of student assessments. Errors in student responses are categorized according to Gagné's taxonomy, highlighting common cognitive challenges. The findings emphasize the need for educational strategies that enhance students' understanding and application of logarithms, aiming to foster their long-term academic success.

Chapter I

INTRODUCTION

1.1. Background of Student Difficulties Understanding Logarithm

Logarithms constitute a fundamental subject within the realm of mathematics, playing a pivotal role in the development of more sophisticated mathematical principles, including exponentials, calculus, and complex numbers. They have a significant impact on multiple academic domains, such as finance, physics, engineering, and computer science, and are frequently employed in problem-solving methodologies within their respective subjects (Nardi, 2000; Tall, 1992). Consequently, logarithms constitute a fundamental component of mathematics courses at the high school and college levels.

However, it is common for students to have challenges when it comes to comprehending and applying logarithms. The obstacles faced by individuals in this context are diverse, involving difficulties in comprehending fundamental principles, employing logarithmic qualities, and comprehending the interrelationships between logarithms and other mathematical ideas, such as exponentials, trigonometry, and calculus (Weber, 2002; Even, 1990; Vinner, 1991).

A study by Dintarini (2018) discusses the difficulties students face in understanding logarithms. To detect the type of errors that students made, author used Gagne's (1984) taxonomy on intellectual skills. Author found that the difficulties are due to a lack of understanding of logarithmic definitions, over-focus on facts of rote and technical procedures, relying on improper intuition, and inconsistencies in symbolic writing and inaccuracy. The challenges may develop due to several circumstances, such as a lack of foundational knowledge (Dintarini, 2018), ineffective instructional techniques (Farkas, 2003), misconceptions, or limited exposure to practical applications of logarithms (Chua & Wood, 2005). It is imperative for educators to get a more profound comprehension of these problems. This not only facilitates the identification of particular areas in which students have the greatest challenges, but also offers valuable insights into the root reasons of these difficulties. With the acquisition of this knowledge, educators have the ability to formulate specific pedagogical approaches and instructional materials that aptly tackle these challenges and facilitate students' understanding of logarithmic principles (Hirsch & Pfeil, 2012).

The previous research indicates that students' struggles with understanding and applying logarithms, potentially imped their progress in more advanced mathematical subjects such as calculus, linear algebra, and differential equations (Dubinsky et al., 2005; Leinhardt et al., 1990). Such struggles can lead to a decrease in confidence, initiating a cycle of mathematical anxiety and avoidance. Therefore, it becomes crucial to identify these challenges early in the learning process, facilitating interventions like individualized tutoring, peer mentoring, supplementary learning materials, or technology-enhanced learning tools to support understanding and retention (Dintarini, 2018; Jenifer et. al., 2023).

1.2.Problem Statement

Identifying and addressing these challenges is essential for fostering students' long-term success in mathematics and related disciplines, at the level of further education. By providing targeted support and interventions early in the learning process, educators can help students build a strong foundation in logarithms, enhance their problem-solving skills, and increase their self-efficacy in mathematics (Ansah, 2016). Effective interventions may include individualized tutoring, peer mentoring (Seery et. al., 2021), supplementary learning materials, or the use of technology-enhanced learning tools to facilitate understanding and retention (Radović et. al., 2019). Additionally, addressing common misconceptions and reinforcing the connections between logarithms and other mathematical concepts can help students develop a more coherent and integrated understanding of mathematics. Furthermore, it is crucial for educators to create a supportive and inclusive learning environment that encourages open communication, collaboration, and a growth mindset (Unlu, 2017). By promoting a positive attitude towards mathematics and fostering resilience in the face of challenges, educators can help students overcome their difficulties with logarithms and empower them to achieve long-term success in mathematics and related disciplines.

The subject of Mathematics in the academic year 2018-2019, examined by the State Examination Centre (SEC) in the Republic of Azerbaijan, provides significant insights into the challenges students face when dealing with logarithms. According to the results, students grappled with tasks related to logarithmic functions and equations (Abbaszadə, Bədəlov, & Şelaginov, 2020; State Examination Centre (SEC), 2020). An example question was given: "Calculate the value of

$$\sqrt{36^{\frac{1}{\log_{12} 6}} + 9^{\frac{1}{\log_5 3}}}$$

According to the properties of logarithms, $\sqrt{36^{\frac{1}{\log_{12} 6}} + 9^{\frac{1}{\log_5 3}}} = \sqrt{36^{\log_6 12} + 9^{\log_3 5}} = \sqrt{6^{2\log_6 12} + 3^{2\log_3 5}} = \sqrt{6^{\log_6 144} + 3^{\log_3 25}} = \sqrt{144 + 25} = 13$ is the correct solution.

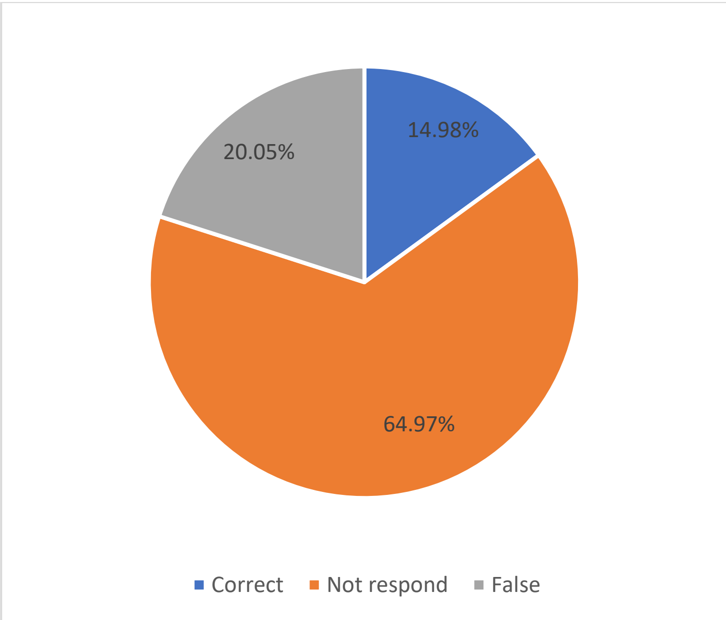

Figure 1: Performance of Students in SEC mathematics exam in 2018 – 2019

Source: Calculated based on data of State Examination Centre (SEC)

Only 14.98% of students answered this question correctly, while a significant 64.97% chose not to respond. The findings suggest that students' difficulties might arise from an insufficient understanding of logarithmic properties and a struggle in executing sequential transformations (M.M. Abbaszadə, T.A.Bədəlov, & O.Y.Shelaginov, 2020; State Examination Centre (SEC), 2020). Such difficulties can hinder students' progress in more advanced mathematical subjects and negatively impact their confidence in their mathematical abilities (Ansah, 2016; Unlu, 2017).

The research problem addressed in this study is the identification and analysis of the specific difficulties that students face in high schools of Baku, the capital of Azerbaijan, when learning logarithms and the potential relationship between their performance on logarithmic tasks and teachers’ perceptions of students understanding and abilities. The problem statement revolves around the notable gap in student performance and understanding of logarithms, as evidenced by the SEC exam results and broader educational observations. Students' challenges are multifaceted, ranging from an insufficient grasp of logarithmic properties and operations to psychological factors such as self-perception and confidence in their mathematical abilities. These issues not only hinder their performance on specific logarithmic tasks but also impede their progression in more advanced mathematical subjects, potentially affecting their overall academic and future career trajectories. To address this problem, the study employed a mixed-methods approach, include quantitative assessments of students' knowledge of logarithms to identify specific areas of difficulty and qualitative interview of teachers to understand the broader context of these challenges. The goal is to gain a comprehensive understanding of the factors contributing to students' difficulties with logarithms and to identify strategies and interventions that can support their learning and improve their performance. Through current research aim is not only to contribute to the academic discourse on mathematics education but also to provide actionable insights for educators, curriculum developers.

1.3. Research Objective and Questions

The primary objectives of this study are to explore the specific difficulties students face when learning logarithms. The second objective is to examine the performance of students on logarithmic tasks and teachers’ perceptions of their understanding and abilities. On other hand, current thesis uses Gagne (1984) Taxonomy to identify error types. To achieve the objectives, the research questions for this study guide the investigation of the specific difficulties that students face when learning logarithms:

1. What are the key difficulties that students encounter when learning and working with logarithms?

1.1. What are the underlying reasons for students' difficulties in understanding logarithms?

2. What teaching methodology and strategies do educators employ to support students learning logarithms?

1.4. Significance of Study

The significance of this study lies in its potential to impact various stakeholders in the educational sphere, from students struggling with logarithms to educators striving for more effective teaching methods. Its findings aim to enhance mathematical understanding, inform educational practices, support student success, address educational equity, guide future research, and contribute to academic discourse. Ultimately, this study serves as a stepping stone toward a more comprehensive understanding of how students learn complex mathematical concepts and how education systems can better support their learning journey.

1.5. Research Structure

The structure for this thesis research is organized into three distinct chapters. The 1^{st} chapter sets the stage for the research by providing a comprehensive introduction to the topic. It establishes the context and significance of the study, focusing on the challenges associated with teaching and learning logarithms. The research objectives and questions are clearly articulated, defining the scope and direction of the study. The chapter also introduces the theoretical underpinning of the research, grounded in Gagné's Intellectual Skills Taxonomy. This framework is essential for analyzing the conceptual challenges students face in grasping logarithmic concepts and understanding the role of educators in this process. Additionally, the chapter discusses the potential impact of technological integration in mathematics education, offering insights into modern pedagogical approaches. In the 2nd chapter, the research design is detailed, describing the mixed-methods approach that combines both qualitative and quantitative research methodologies. It delineates the processes of data collection, including the selection criteria for participating teachers and students, the typeof teacher interviews, and the design of student assessment tasks in logarithms. The chapter further explains the data analysis strategies, encompassing thematic analysis for qualitative data from teacher interviews and descriptive statistical analysis for quantitative student assessment data. This section ensures that the research methods are transparent and replicable, providing a robust foundation for the study's findings. The final 3rd chapter presents the findings of the study, beginning with an analysis of the teacher interviews that explores educators' perspectives on the difficulties students encounter in learning logarithms. It then delves into the analysis of student assessment data, highlighting common areas of struggle and error patterns in logarithmic tasks. The chapter proceeds to discuss these findings in relation to the research questions and theoretical framework, drawing connections between teachers' perceptions, student performance, and Gagné's taxonomy. The chapter concludes with discussion by acknowledging the limitations of the current Capstone project and proposing avenues for future research. It underscores the potential enhancements and adaptations that the educational product may undergo, based on evolving educational needs and feedback from its practical implementation. This forward-looking perspective ensures that the Capstone project remains a dynamic, evolving solution, aimed at continually improving students' comprehension and mastery of logarithm. On other hand, the appendices provide supplementary materials that support the research that includes the full set of interview questions used with teachers and the specific logarithm assessment questions administered to students, offering additional context and depth to the research methodology and findings.

Chapter II

LITERATURE REVIEW

2.1. The Concept of Logarithms

The origin of logarithms may be traced back to the early 17th century, when John Napier (1614), a mathematician from Scotland, introduced them as a means to streamline intricate computations (Whiteside, 2014). Logarithms have emerged as a fundamental component of mathematical theory, finding widespread utilisation across diverse disciplines including physics, engineering, computer science, and finance. Napier's early research on logarithms entailed using the reciprocal connection between multiplication and exponentiation to construct a tabular representation that streamlined the execution of intricate multiplication and division operations (Whiteside, 2014). Following Napier's initial discovery, the mathematician Henry Briggs (1624) made significant contributions by introducing a base-10 logarithm system referred to as common logarithms. This development served to enhance the understanding and practical applications of logarithms (Whiteside, 2014).

Logarithms are useful mathematical concepts that can be applied in a wide variety of contexts. In the field of finance, they are utilised in the calculation of compound interest as well as the modelling of exponential development or decay (Kellison, 2009). Logarithmic scales are used in the fields of physics and engineering to measure quantities with a wide range of magnitudes (Feynman et al., 2011). Some examples of logarithmic scales include the decibel scale and the Richter scale. According to Corden et al. (2009), logarithms are essential tools for computer scientists when working with algorithms, data structures, and complexity analysis. In spite of the significance of logarithms, students frequently experience frustration when attempting to master them. These obstacles can include gaining a comprehension of the fundamental ideas, using logarithmic properties, comprehending the links between logarithms and other mathematical ideas, and visually representing logarithmic functions and their graphs (Weber, 2002; Even, 1990; Vinner, 1991). It has been demonstrated through research conducted by Leinhardt et al. (1990) and Dubinsky et al. (2005) that students' struggles with logarithms can impede their progression in more advanced mathematical courses and have a detrimental impact on their confidence in their mathematical ability.

Various pedagogical approaches have been proposed to address the challenges students face when learning logarithms. Some researchers have suggested incorporating real-life examples and applications of logarithms to make the concepts more relatable and engaging (Kadijevich, 2000). Others have recommended using visual representations, such as graphs and number lines, to enhance students' understanding of the relationships between logarithmic functions and their properties (Nardi, 2000). Additionally, technology, including graphing calculators and computer-based learning environments, has been shown to be beneficial for teaching logarithms and improving students' comprehension of the subject (Tall, 1992).

The concept of logarithms is an essential topic within mathematics, with wide-ranging applications across various fields. Despite the inherent challenges students face when learning logarithms, research has provided valuable insights into effective pedagogical approaches that can help students overcome these difficulties and gain a deeper understanding of logarithmic concepts. Continued research on teaching and learning logarithms will contribute to the development of more effective instructional strategies, ultimately promoting students' success in mathematics and related disciplines.

Future research on the teaching and learning of logarithms could focus on exploring the effectiveness of various instructional strategies, such as inquiry-based learning, problem-based learning, or flipped classroom models, in addressing students' difficulties with logarithms (Artigue & Blomhøj, 2013; Prince & Felder, 2006). Additionally, researchers could investigate the role of technology, including computer simulations, virtual manipulatives, and educational software, in enhancing students' understanding of logarithmic concepts and their applications (Hohenwarter et al., 2008). Assessment plays a crucial role in the learning process, as it helps educators identify students' strengths and weaknesses and adjust their teaching strategies accordingly (Black & Wiliam, 1998).

Research on the assessment of logarithm learning could focus on developing diagnostic tools to identify specific difficulties students face when learning logarithms and track their progress over time. Moreover, researchers could explore the use of formative assessment techniques, such as feedback, self-assessment, and peer assessment, in fostering students' understanding of logarithmic concepts (Nicol & Macfarlane-Dick, 2006).

Affective factors, such as motivation, self-efficacy, and anxiety, play a significant role in students' learning experiences and outcomes (Pajares, 1996). Research on logarithm learning could investigate the impact of these affective factors on students' understanding of logarithmic concepts and explore strategies to address these factors, such as creating a supportive learning environment, promoting a growth mindset, or incorporating metacognitive strategies into logarithm instruction (Dweck, 2006; Schraw et al., 2006).

2.2. Difficulties in Learning Logarithms

2.2.1. The Struggle with Abstract Concepts

The difficulty in understanding abstract mathematical concepts is widely discussed in the existing literature. Mirzəzadə M.N. (2023), in the work "Some Problems of the Formalization of Mathematical Analysis Concepts," delves into the struggles students in Azerbaijan face in comprehending abstract mathematical concepts such as those encountered in mathematical analysis. These struggles become particularly evident when students attempt to grasp the concept of a logarithm, which requires a certain degree of abstract thinking. Mirzəzadə M.N.'s work underscores the importance of providing concrete examples when teaching such abstract mathematical concepts, as tangible real-world examples can serve to make the abstract nature of logarithms more understandable to students. Səlimli Z.V. (2023) in the text "Mathematical Methods as a Means of Mathematization of Sciences" underscores the relevance of mathematical concepts across a broad spectrum of disciplines. This is pertinent to understanding logarithms, as they are widely used in many fields, including physics, engineering, and computer science. Moreover, they are even used in the calculation of sound intensity levels in decibels in acoustics. Thus, ensuring a strong understanding of logarithms not only serves students within the classroom but also in their future professional endeavours across various scientific disciplines. Mayer, R.E. (2004) argues that the most effective method of instruction is one that strikes a healthy balance between the presentation of abstract theory and concrete instances. This is in line with the emphasis that Mirzəzadə (2023) places on using concrete examples when teaching abstract ideas like logarithms, and it works well with that philosophy. Boaler, J. (2016) discusses the significance of teaching students to have a growth mindset in order to assist them in overcoming obstacles that arise when attempting to comprehend abstract ideas. This mental framework allows students to regard struggle as a natural part of the learning process, which is particularly advantageous when students are attempting to grapple with abstract concepts such as logarithms. The article by Walle, (2013) discusses the significance of using visual aids and manipulatives while instructing students on abstract mathematical ideas. Students who are having trouble visualising logarithmic scales and the relationships between numbers may benefit from using graphing calculators, software, or even just plain number lines.

2.2.2. Educational Strategies and The Role of Teachers

The study by Fərmanlı Ü.İ. (2021) highlights important themes in the field of mathematics education, particularly in relation to practical applications and cross-disciplinary connections. The author emphasizes the need to modernize teaching methodologies to align with curriculum standards and prepare students for real-life situations. The practical application of mathematical knowledge is seen as crucial, and the author suggests incorporating measures in the curriculum to develop students' practical skills. The role of the teacher is emphasized in connecting learning with real-life examples. The findings underscore the need for practical application, teacher guidance, cross-disciplinary connections, and problem-solving in mathematics education. Understanding these difficulties will inform the development of effective educational strategies and approaches for facilitating students' comprehension of logarithmic concepts.

Furthermore, the transformative role of Information and Communication Technologies (ICT) in mathematics education is highlighted by Cəlilova V. M. (2018). The use of ICT tools in teaching mathematical concepts introduces innovative elements that can enhance learning outcomes, capture student interest, and facilitate teachers' lesson preparation. These tools can bring an abstract concept like logarithms to life, making it more accessible and engaging for students. Interactive teaching methods leveraging ICT and taking into account individual student characteristics can yield superior results and may serve as an effective strategy to help students overcome the challenges of understanding logarithms.

Mayer (2004) suggests a cautious approach to discovery-based learning, highlighting the importance of structured guidance for effective teaching. In the context of teaching logarithms, teachers might introduce the discovery process through well-designed activities that guide students to uncover the relationship between exponential and logarithmic functions. Mayer's (2004) skepticism about "pure discovery learning" underlines the importance of combining discovery methods with explicit instruction, particularly for abstract concepts like logarithms.

Jo Boaler's research (2002) explores the impact of various teaching methodologies, contrasting traditional and reform-based approaches. Traditional methods might involve rote memorization of logarithmic properties and problem-solving through repeated practice. Reform methods could incorporate real-world applications of logarithms, making them relatable and functional. Boaler suggests that a balanced approach, integrating both traditional and reform methods, can result in the most effective mathematics education.

Hiebert & Grouws (2007) delve into how the classroom environment affects mathematical learning. Teachers play a pivotal role in creating a conducive environment for grasping complex concepts like logarithms. Employing interactive techniques and creating a feedback-rich environment can be instrumental. Moreover, teachers can adapt their strategies based on continuous assessment, thereby directly affecting students' learning outcomes.

Drijvers, Kieran, and Mariotti (2010) discuss the theoretical perspectives on integrating technology into mathematics education. In the realm of logarithms, employing technological tools like graphing calculators or interactive software can significantly aid the learning process. These tools can help visualize logarithmic functions, make connections with exponential functions, and solve real-world problems that involve logarithms. Furthermore, teachers can leverage these technologies to customize lessons and assessments, adapting to the individual needs of the students.

Moreover, the techniques may include Experiential Learning (Prince & Felder, 2006), where students can be engaged in simulations or problem-solving exercises that enhance their understanding of logarithmic functions. Visual Representation (Nardi, 2000; Tall, 1992) can also be employed, offering graphical representations of logarithmic functions that clarify abstract concepts. Furthermore, the integration of technology (Hohenwarter et al., 2008) can facilitate interactive learning experiences, such as using software tools to visualize and manipulate logarithmic graphs. This dual focus on understanding and remediation hopes to foster an enhanced learning experience for students grappling with logarithms, thereby contributing to the broader body of knowledge on the teaching and learning of logarithms.

The incorporation of educational technology, particularly graphing calculators such as Desmos, is a crucial factor in fostering student engagement and facilitating the acquisition of mathematical concepts (Karadeniz & Thompson, 2017; Liang, 2016). These technologies function as versatile instruments that foster students' engagement with mathematical topics, promote critical thinking, and urge them to scrutinise their own comprehension as well as the functionality of the technology itself. There is a growing recognition among educators of the need of integrating these tools into their teaching methods, as they offer interactive means of representing and investigating mathematical ideas (Karadeniz & Thompson, 2017).

The section highlights the intricate nature of instructing and comprehending mathematical functions. It is not uncommon for students to encounter difficulties in comprehending the intricacies associated with various types of functions, such as exponential and logarithmic functions (Kenney & Kastberg, 2013; Weber, 2002a). Conventional pedagogical approaches may not comprehensively address these impediments to learning. The challenge becomes more pronounced when working with exponential and logarithmic functions because of their distinct notational and operational properties (Ural, 2017; Weber, 2002a).

Educational technologies, such as Desmos, have the potential to exert a substantial influence on the teaching and learning processes. In addition to facilitating the plotting of mathematical functions, Desmos offers a variety of pre-designed exercises that can be accessed and utilised for educational training (Thomas, 2015). The impact of Desmos and related technologies on students' comprehension of functions, as well as their ability to apply them in real-world scenarios and engage in problem-solving, has been supported by empirical study conducted by Oates, Sheryn, and Thomas (2014), Liang (2016), and Hollar and Norwood (1999). The comprehension of functions, particularly the reciprocal connections between exponential and logarithmic functions, has been recognised as a domain in which students have difficulties (Weber, 2002a; Ural, 2006). Nevertheless, the utilisation of instructional technology has the potential to substantially mitigate these difficulties. The utilisation of tools such as Desmos can facilitate the active engagement of students in the creation, manipulation, and comprehension of functions within an interactive setting, hence supporting the internalisation of intricate mathematical concepts (King, 2017; Liang, 2016).

In summary, the incorporation of instructional technologies into the implementation of curriculum is deemed essential. It is vital for educators to possess knowledge regarding the range of technology resources at their disposal and comprehend how these resources might be effectively utilised to enhance students' comprehension of complex subjects, such as functions (Hoang & Caverly, 2003; Venturini, 2015).

2.2.3. Challenges in Teaching of Logarithms

Logarithms, forming an integral component of high school and college mathematics curricula, are foundational to understanding more advanced mathematical concepts and find practical applications in various disciplines. Despite their significance, students often encounter challenges when learning logarithms (Weber, 2002; Even, 1990; Vinner, 1991; Leinhardt et al., 1990; Dubinsky et al., 2005).

1) Grasping the basic concept of logarithms: Students may have a difficult time understanding the fundamental idea that logarithms are the inverses of exponentiation and how logarithmic functions relate to exponential functions (Nardi, 2000).

   Example: To understand that $\boldsymbol{log_a b = c \Leftrightarrow a^c = b}$

2) Applying logarithm properties: Students may have trouble remembering and applying the properties of logarithms, such as the product rule, quotient rule, power rule, and change of base formula (Dreyfus & Eisenberg, 1986). This can be a challenge for students because the product rule, quotient rule, power rule, and change of base formula are all mathematical rules.

   Example: Use the product rule $\boldsymbol{log\,(a * b) = log\,(a) + log\,(b)}$ *to simplify* $\boldsymbol{log\,(15)}$ *into* $\boldsymbol{log\,(3) + log\,(5)}$

3) Solving logarithmic equations and inequalities': Students may find it difficult to handle and solve equations and inequalities using logarithms, particularly when many logarithmic terms or bases are involved (Weber, 2002). This is especially true when students are asked to solve equations and inequalities containing exponential functions.

   Example: Solve $\boldsymbol{log\,(x) + log\,(x-2) = 1}$ *. Combine logs and convert to exponential form to solve for x.*

4) Converting between logarithmic and exponential forms': Students may have difficulty converting logarithmic equations to exponential equations and vice versa, which is a

crucial skill for solving a variety of logarithmic and exponential issues (Dubinsky et al., 2005). Logarithmic and exponential problems can be broken down into two categories: linear and nonlinear.

Example: Convert $\boldsymbol{log_3 9 = 2}$ *to exponential form* $\boldsymbol{3^2 = 9}$

5) Working with different bases: According to Even (1990), students could have trouble comprehending the difference between natural logarithms (which use base e) and common logarithms (which use base 10) as well as how to work with logarithms that use other bases.

   Example: Convert $\boldsymbol{ln\,(2)}$ *to* $\boldsymbol{log_{10} 2}$ *using* $\boldsymbol{ln\,(2) = \frac{log_{10} 2}{log_{10} e}}$

6) Understanding real-life applications: Students may have difficulty understanding how logarithms are utilised in a variety of real-life applications, such as in finance, physics, engineering, and computer science (De Bock et al., 2000). This concept is covered in number six of this section.

   Example: In finance, compound interest $\boldsymbol{A = P(1 + \frac{r}{n})^{nt}}$ *can be rearranged using logarithms to solve for time t.*

7) Visualising logarithmic functions and their graphs: Students may have difficulty understanding the graphical representation of logarithmic functions, including their shapes, asymptotes, and transformations, as well as how they relate to exponential functions (Vinner, 1991). This is because students may have difficulty visualising logarithmic functions.

   Example: $\boldsymbol{y = log\,(x)}$ *will have a vertical asymptote at x = 0 and pass through the point (1,0).*

8) Navigating notation and terminology: Students may have trouble with the notation and terminology used in logarithms, such as comprehending the distinction between log and ln, or getting a handle on the concept of base and exponent (Leinhardt et al., 1990). Logarithms are utilised in a variety of fields, including physics, chemistry, and engineering.

Example: Understand that $\boldsymbol{ln}(\boldsymbol{x})$ *is* $\boldsymbol{log}_{\boldsymbol{e}}(\boldsymbol{x})$*, and* $\boldsymbol{log}\ (\boldsymbol{x})$ *typically means* $\boldsymbol{log}_{\boldsymbol{10}}(\boldsymbol{x})$ *in common usage.*

9) Simplifying complex logarithmic expressions: Students may find it tough to simplify and evaluate complex logarithmic expressions, particularly when several logarithmic properties need to be applied in a certain order (Byerley et al., 2015). This is because students must apply the logarithmic qualities in the order that they are listed in the expression.

   Example: Using the quotient rule such as $\boldsymbol{log}\ (\boldsymbol{a}/\boldsymbol{b}) = \boldsymbol{log}\ (\boldsymbol{a}) + \boldsymbol{log}\ (\boldsymbol{c})$

10) Connecting logarithms to other mathematical concepts: Students may have difficulty seeing the connections between logarithms and other mathematical concepts, such as exponentials, trigonometry, or calculus. This can hinder their overall understanding as well as their ability to solve more advanced problems (Confrey & Smith, 1994).

   Example: In calculus, $\int \frac{\boldsymbol{1}}{\boldsymbol{x}} \boldsymbol{dx} = \boldsymbol{ln}|\boldsymbol{x}| + \boldsymbol{C}$*, showing the link between logarithms and integrals*

The literature on students' difficulties when learning logarithms highlights several common challenges, ranging from grasping basic concepts to connecting logarithms with other mathematical areas. Identifying these difficulties is essential for developing targeted teaching strategies and resources to address these issues and support students in their understanding of logarithmic concepts.

2.3. Theoretical and Conceptual Framework

2.3.1. Role and Types of Assessments in Logarithmic Learning

Assessment and evaluation play crucial roles in education, particularly in understanding student performance and informing instructional strategies (Black & Wiliam, 1998). In the context of teaching and learning logarithms, effective assessment and evaluation methods can provide valuable insights into students' understanding of logarithmic concepts, identify specific difficulties they may be facing, and help educators tailor their instruction to address these challenges.

Traditional Assessment Methods

Traditional methods of assessment in logarithm learning often involve the use of paper-and-pencil tests, including multiple-choice, short-answer, and problem-solving questions (Nicol & Macfarlane-Dick, 2006).

Formative Assessment Techniques

In the context of logarithm learning, formative assessment may involve the use of questioning strategies, self-assessment, peer assessment, and classroom discussions to gauge students' understanding of logarithmic concepts and identify misconceptions or challenges they may be facing (Nicol & Macfarlane-Dick, 2006).

Diagnostic Assessment Tools

In the context of logarithm learning, diagnostic assessments may involve the use of computer-based systems, such as intelligent tutoring systems or adaptive learning platforms, which can adapt to students' individual needs and provide personalized feedback on their understanding of logarithmic concepts (Hohenwarter et al., 2008).

Performance-based Assessment

In the context of logarithm learning, performance-based assessment may involve the use of problem-solving tasks, projects, or investigations that require students to apply their understanding of logarithmic concepts to real-life scenarios, such as financial calculations, scientific measurements, or engineering applications (Kadijevich, 2000).

The assessment and evaluation of logarithmic understanding are critical components of the teaching and learning process. A variety of assessment methods, including traditional, formative, diagnostic, performance-based, alternative, and technology-enhanced assessment, can provide valuable insights into students' understanding of logarithmic concepts and inform the development of effective instructional strategies. By incorporating metacognitive strategies into assessment practices, educators can further promote students' understanding and success in logarithm learning. Continued research on the assessment and evaluation of logarithmic understanding will contribute to the development of more effective assessment tools and strategies, ultimately enhancing students' learning experiences and outcomes in mathematics and related disciplines.

2.3.2. Theoretical Framework: Gagné's Intellectual Skills Taxonomy

According to Gagné's taxonomy of learning, there are five distinct categories of capacities that can be acquired through education: intellectual skills, cognitive strategies, linguistic knowledge, attitudes, and motor skills (Gagné & Briggs, 1974; Gagne, 1984). In the field of instructional design, this framework is held in very high respect, particularly for its capacity to differentiate between abstract and concrete forms of learning (Reigeluth, 1983; Seels & Glasgow, 1990).

1) Motor Skills: The ability to do physical acts such as starting a car or swinging a golf club is an example of a motor skill.
2) Attitude: An individual's internal emotional state, known as their attitude, can have a significant impact on the choices they make on their behaviour, such as whether or not they go to an art gallery.
3) Verbal Information: Information Provided Verbally Can Be Divided Into Two Categories:
    i. Labels and Facts: The process of naming something or answering verbally or in writing to specific input, such as naming the capital cities of several countries, is referred to as labels and facts.
    ii. Bodies of Knowledge: The ability to recall a wide and interrelated set of facts, such as rephrasing written material or reciting rules and regulations, is required for bodies of knowledge.
4) Cognitive Strategy: The term "cognitive strategy" refers to the internal processes that enable an individual to take control of their own learning. Some examples of cognitive

strategies include conducting self-tests and asking pertinent questions in order to acquire new knowledge.

5) Intellectual Capabilities are comprised of the following four sub-levels:
    a. Discrimination: Discrimination is the process of differentiating one member of a class from another, for as differentiating between different species of birds based on their markings.
    b. Concrete Concept: The act of reacting in a comparable manner to a group of things or events, such as dividing music into its various genres, is an example of a concrete concept.
    c. Rule Using: It is refers to the process of applying distinct rules to various scenarios, which are frequently conceived of as "if-then" expressions.
    d. Problem Solving: Combining lower-level rules to solve higher-level problems and maybe developing new rules through trial and error are both examples of problem solving.

(Gagne, 1984)

The theoretical framework for this research is rooted in an adapted version of Robert Gagné's Intellectual Skills taxonomy, tailored specifically to address the complexities and learning challenges associated with logarithms. This adaptation, as outlined by Dintarini (2018), integrates Gagné's framework with terminologies and concepts central to the subject of logarithms—facts, concepts, principles, and skills. This integration provides a nuanced perspective for examining and understanding students' difficulties with logarithms. In this adapted model, Gagné's taxonomy is tailored to encapsulate the unique cognitive processes involved in learning logarithms: .

A. Discrimination

At the level of Discrimination, students are expected to recognize and differentiate between foundational facts and symbols in logarithmic expressions. For instance, students should distinguish the notation $\boldsymbol{y = log_a x}$ as the mathematical inverse of $\boldsymbol{y = a^x}$

B. Concrete Concepts

This domain necessitates a comprehension of the abstract constructs in logarithmic functions, epitomized by the equivalence $\boldsymbol{log_a b = c \Leftrightarrow a^c = b}$

C. Rule Using

Students at this level must be proficient in manipulating logarithmic principles, represented by the theorem $\boldsymbol{log_a bc = log_a b + log_a c}$

D. Problem Solving

This advanced tier involves the synthesis of skills, concepts, and principles to decipher intricate logarithmic equations or to apply logarithms in authentic scenarios. For instance, resolving the equation $log_2(x + 1) = 3$ to find x. Or another sample, applying logarithmic principles to calculate compound interest for an initial sum of $1000 at a 5% annual rate over 5 years.

By coupling Gagné's Intellectual Skills taxonomy (Gagné & Briggs, 1974; Gagne, 1984) with an elaborative breakdown of logarithmic facts, concepts, principles, and skills, the framework aims to furnish an academically rigorous methodology for diagnosing and ameliorating student difficulties in understanding logarithms.

Chapter III

METHODOLOGY

3.1. Research Design and Sampling

The aim of this thesis research to explore the specific challenges Azerbaijani high school students encounter in understanding logarithms. By analysing teacher perspectives and student performance, the study seeks to uncover key areas of difficulty in logarithmic concepts. The objective is to enhance teaching methods and improve learning outcomes in this critical area of mathematics education. The study investigates several research questions: What teaching methodology and strategies do educators employ to support students learning logarithms? What are the common difficulties students face in understanding logarithms? And What is the reason of this difficulties that students challenges when understanding logarithms? These questions are designed to explore both the instructional approaches and the student learning experience in depth. The research involves a mixed-methods approach, engaging both qualitative and quantitative methods. Participants include five teachers from different schools and a group of 57 high school students. This sample size students can provide a reasonable representation of the target population, especially when the population is large as whole country students and diverse as whole high school students in Azerbaijan. Sometimes, practical limitations such as time, budget, or availability of participants make it challenging to have a larger sample size. In such cases, a sample size of 50 - 60 could be justifiable as it balances the need for meaningful results with available resources (Lakens, D., 2022)

The qualitative data from teacher interviews undergo thematic analysis to identify common pedagogical approaches and perceived student difficulties. The student assessment data is quantitatively analyzed to identify patterns in performance and common errors. These errors are categorized according to Gagné's (1984) taxonomy, focusing on areas where students frequently answered incorrectly or left questions blank.

3.2. Data Collection and Sources

The data collection process is bifurcated into two main streams: qualitative data from teacher interviews and quantitative data from student assessments. The data collection and sampling for the research were conducted with careful consideration of the academic schedule and school structure. The choice to target 11th graders instead of 10th graders, despite logarithms being part of the 10th-grade curriculum, was due to the timing of the research. As the research was conducted in the first term, and logarithms were taught in the second term, 11th graders who had already covered the topic were chosen as participants.

One of the schools where the research was conducted is a secondary public school located in the Narimanov district of Baku. I investigated the Ministry of Education's website to see whether any schools had students that actively participated in the ministry's events and programs. This institution caught my eye and gave me the impression that it would be eager to take part in my study project. Their encouragement and involvement did, in fact, make the atmosphere conducive to gathering data and increased the level of cooperation and engagement among the participants. For this reason, it is taken as an example. At the same time, the reason for conducting the research in this school is that there are no special subject specified classes. (In some of the secondary schools, classes are split based on students' interests in different subjects beginning in the 9th grade). That is, the classes out of 4 were chosen randomly. (29 students passed the test). The other school, catering exclusively to 10th and 11th graders, focuses on preparing students for complete (11-year) secondary education level exam of SEC (State Exam Center). with stratified classes based on prior academic performance. Students were randomly selected from each level— A (80-90), B(70-80) and C (60-70) classes—to guarantee data validity and mitigate any selection bias. The selection of teachers for interviews was constrained by time limitations and their availability amidst teaching schedules. Despite these constraints, a comprehensive range of grade teachers, from 5th to 11th grade, were incorporated into the interview process to gather diverse educational insights and pedagogical perspectives. These teachers currently teaching middle year students (from 5th to the 9th grade) also bring previous experience teaching logarithms to 11th graders.

The selection criteria aim to ensure a diverse representation of educational backgrounds and teaching experiences. Semi-structured interviews are conducted, allowing for both guided questions and open-ended responses. The interviews are designed to delve into the teachers' perceptions of students' difficulties with logarithms, their teaching methodologies, and their views on the effectiveness of current teaching materials. The interview questions cover a range of topics, including:

- Teachers' experiences and background in teaching mathematics.
- Strategies used to identify and address students' difficulties in learning logarithms.
- Perspectives on the effectiveness of teaching materials and methodologies.
- Observations on common student challenges with logarithmic concepts and tasks.
- Suggestions for enhancing student understanding and engagement with logarithms.

Interviews are recorded with the consent of the participants and later transcribed for analysis (Appendixes A and Appendixes B). This ensures accuracy in capturing the responses and facilitates thorough analysis. On other hand, a group of 57 high school students are selected to participate in the assessment component of the study. The students are given a set of 15 logarithmic questions (Appendixes C), curated from SEC materials and organized into three difficulty categories: Easy, Medium, and Difficult.

Students' responses to the logarithmic questions are collected and documented The focus is on both correct and incorrect answers, as well as questions left unanswered, to gain a comprehensive understanding of their proficiency and areas of difficulty.

Finally, the incorrect and incomplete responses from the student assessments were analysed and categorized based on Gagné's (1984) taxonomy of learning (Discrimination, Concrete Concepts, Rule Using, and Problem Solving). Each incorrectly solved question is coded according to the predominant type of error, providing insights into the specific cognitive challenges faced by the students in understanding and applying logarithmic concepts.

3.3. Data Analysis Methods

The data analysis for this research project is designed to comprehensively interpret the qualitative data from teacher interviews and the quantitative data from student assessments. The transcribed interviews with teachers have been undergo thematic analysis, a robust qualitative-quantitative mix method for identifying and interpreting patterns within data. This process involves several stages, starting with a thorough familiarization with the data to gain an in-depth understanding of the content.

The student assessment data have been analyzed using descriptive statistical methods via MS Office Excel software. It involves calculating the percentages of correct, incorrect, and unanswered responses for each logarithmic question and across different difficulty levels. In addition, each incorrectly answered or attempted question have been categorized according to Gagné's (1984) taxonomy of intellectual skills. This categorization helps identify the predominant types of errors students make, such as issues with Discrimination, Concrete Concepts, Rule Using, or Problem Solving. The frequency of each error type has been quantified to reveal common patterns in students' understanding and application of logarithmic concepts. Finally, ‘‘Logarithm assessment questions for students’’ was evaluated utilizing Traditional Assessment method - use of paper-and-pencil tests, including multiple-choice and problem solving questions (Nicol & Macfarlane-Dick, 2006). The extreme challenged questions that students faced were reviewed and analysed from pedagogical perspective by exam maker.

Limitations of Research

The small sample size of five pedagogues may not adequately represent the broader community of mathematics teachers. The qualitative aspect of data introduces subjectivity, as it's based on personal perceptions and interpretations of these teachers. The error analysis is limited to specific questions and categories and doesn't account for individual variations or the complete spectrum of probable misunderstandings. The study lacks a longitudinal perspective to examine the progress of students' comprehension over time and doesn't incorporate direct input from the students themselves. It relies on self-reported data from teachers, which could not comprehensively capture actual classroom dynamics or student performance. Additionally, the study doesn't comprehensively explore extrinsic elements like socioeconomic background or psychological components that could influence learning. These limitations emphasise the need for a larger, more comprehensive and quantitative study in the future to better identify and address students' difficulties with logarithms.

Chapter IV

RESULTS OF ANALYSIS

4.1.Pedagogical Perspectives on Students Difficulties Understanding Logarithm

The interview questions (Appendixes 1) are designed to gather qualitative data from mathematics teachers (pedagogues) regarding their perceptions of students' difficulties with logarithms. Each question seeks to probe different aspects of the instructional environment, the teachers' pedagogical approaches, and their understanding of the specific challenges students encounter with logarithms. The questions explore the teachers' experience and methods for identifying and addressing students' difficulties with logarithms, the effectiveness of current teaching materials and methodologies, and students' grasp of key concepts such as the relationship between exponents and logarithms, as well as their ability to apply fundamental logarithmic properties. They also seek teachers' opinions on why students struggle with logarithms and their recommendations for instructional strategies that could enhance students' understanding and mastery of the topic.

Pedagogue	Experience	Grade
1	*15 years*	*Pedagogue, Math Instructor of 5th, 6th and 8th Grades*
2	*45 years*	*Pedagogue, Former Principal, Awarded with an Honorary Order*
3	*10 years*	*Pedagogue, Math Instructor of 10th and 11th Grades*
4	*6 years*	*Pedagogue, Math Instructor of 9th, 10th and 11th Grades*
5	*30 years*	*Pedagogue, Math Insturctor of 10th and 11th Grades*

Table 1: Background description of Pedagogues

Source: Calculated by author

The responses from the five pedagogues provide a snapshot of their teaching experience in mathematics, ranging from 6 to 45 years. Pedagogue 1 has a solid 15 years of teaching in Baku, suggesting a deep familiarity with the local educational landscape. Pedagogue 2, with an extensive 45-year career, has also held a principal position and has been recognized by the state for their contributions to education, indicating a high level of expertise and respect in the field. Pedagogue 3, with a decade of experience, may bring more recent educational strategies into their practice. Pedagogue 3's potential to introduce modern strategies is based on the premise that recent teaching experiences often align with current educational innovations. Pedagogue 4, with 6 years of experience, mentions teaching in both local and international institutions, suggesting exposure to diverse educational systems. Pedagogue 5 has a significant 30 years of experience and has worked in various schools, likely offering a broad perspective on mathematics education. Collectively, these teachers bring a wealth of knowledge and a variety of pedagogical approaches to the subject of mathematics.

Students Challenges

- *Students Do Not Master the subject of Logarithms well*

Teacher responses show a range of logarithm proficiency experiences. Pedagogue 1 thinks that logarithms may be part of a larger maths problem. The teacher suggests that a student's logarithm ability is indicative of their mathematical understanding since individuals who struggle with basic number concepts struggle with trigonometry and logarithms. Pedagogue 2 has seen students understand logarithmic ideas faster, demonstrating that students learn differently. This teacher believes students can grasp logarithms faster. Logarithms are complicated, and Pedagogue 3 notes that students either forget or memorise without comprehending. Logarithms' intricacy and various subtopics require repeated exposure; hence they recommend integrating them with other mathematical courses to improve retention and understanding. Logarithms are attractive and approachable to Pedagogue 4, who is startled by the need for investigation into their challenges. Despite their math weaknesses, students can tackle simple logarithmic questions and approach the issue with passion. Pedagogue 5 knows logarithms may be difficult, but with the correct instruction, pupils can understand them. He recommends that teachers should not cover logarithms from the beginning to the end, but should teach logarithms in parts and return to logarithms several times during the year. This is because students need time to understand and remember the subject well. The school team and the education system play a major role in this. Students need instructional support to master logarithms, as this response shows.

- *Common Problem – Less Understanding the Fundamental Mathematic Concepts*

The teachers discuss many logarithm challenges students experience. Students struggle with formula-based logarithmic expressions and equations, especially when using them in instances, according to Pedagogue 1. Pedagogue 2 emphasises the significance of a good mathematical foundation by stating that basic concerns such not knowing the multiplication table can lead to exponentiation and logarithm problems. Pedagogue 3 says that if students struggle with exponentiation, they won't understand logarithms, hence they must master related concepts. After some effort, students can reconcile logarithmic equations and inequalities, according to Pedagogue 4. Pedagogue 5 emphasises the difficulty students encounter in applying logarithmic ideas to real-world problems owing to their abstract character.

- *Main Reasons of Common Problem*

The teachers give several causes for students' logarithm problems. Mathematical ability influences comprehension, according to Pedagogue 1. Some students are naturally bright, while others, especially humanities students, may struggle with maths. According to Pedagogue 2, poor early teaching in simple concepts like the multiplication table can lead to problems with more sophisticated topics like logarithms. This means that this problem is a widespread. The intricacy and duration of the 10th and 11th-grade mathematics curriculum might confuse pupils, especially when they fail to distinguish between logarithms, trigonometry, and inequalities, according to Pedagogue 3. Pedagogue 4 emphasises the need of integrating concepts, emphasising that logarithms must be connected to other mathematical characteristics and that students without the wider equation-solving abilities needed struggle. Logarithms are difficult to learn due to a lack of fundamental comprehension, fear of the subject's complexity, unfamiliarity with new concepts, and motivation, according to Pedagogue 5. They also indicate that students don't understand math's practical applications, which affects motivation and learning.

- *Individual Aptitudes in Understanding the relationship between Exponential and Logarithmic equations*

The pedagogues asked opinion about that can students easily see the relationship between exponential and logarithmic equations such as $a^y = x$ and $log_a x = y$ and build them fluently? More specifically, do they know that $a^y = x$ and $log_a x = y$ are equivalent notations? Teachers evaluated students' exponential-logarithmic equation comprehension and translation. Pedagogue 1 says students can communicate across notations easily. Some students become confused when applying logarithms as an alternative representation of superscript numbers. Based on Pedagogue 2, a student who knows exponentials and superfunctions can readily link the logarithmic expression, suggesting that basic knowledge helps students grasp. According to Pedagogue 3, some students can recognise the link between the notations, while others need to be reminded of the superscript function. Pedagogue 4 argues that students from "advanced " high school (schools with high results in the State Entrance Exam of Azerbaijan) can compute the power column and understand logarithms when introduced, demonstrating that past knowledge affects their understanding. Pedagogue 5 stresses the equivalence of both notations but acknowledges that students' capacity to draw these connections varies. Some grasp instinctively, while others need reinforcement, suggesting math skills may impact this knowledge.

- *Degree of Proficiency of Fundamental Mathematical Concepts*

The responses from the pedagogues regarding students' abilities to master and apply the fundamental properties of logarithms such as the product rule ($log_c xy = log_c x + log_c y$).

Pedagogue 1 thinks students comprehend properties well but struggle with complicated equations, especially logarithmic inequalities, since they don't know how to manage the base when it's less than one. Pedagogue 2 observes fundamental difficulty in constructing logarithmic equations and inequalities but worries more about students' abilities to solve equations and discover the unknown, especially in inequalities. Pedagogue 3 says student mastery varies. To minimise long-term misunderstanding and student hate, they teach logarithmic properties with parameters, not numbers. They also note that students can easily prove logarithmic characteristics with examples. Students know logarithmic characteristics but struggle to apply them, according to Pedagogue 4. Reminding students of the qualities helps but applying them to more complicated expressions is difficult. Pedagogue 5 compares logarithms to a domino effect where early problems escalate to more. They think that logarithmic characteristics are innately linked to mathematical capability and that students who learn the basics may apply them.

Pedagogical Strategies and Support

Teachers utilise several diagnostic approaches to teach math. Pedagogue 1 looks in students for inability to apply mathematical principles in real life to highlight challenges, whereas Pedagogue 2 employs post-lesson questioning to detect knowledge gaps. Students' mathematical thinking is assessed with early evaluations and follow-up questions by Pedagogue 3, while Pedagogues 4 and 5 highlight continuing assessments, observational tactics, and open communication channels for students to voice their challenges. The effectiveness of teaching materials and approaches divides teachers. According to Pedagogue 1, textbooks and class exams prepare students for math. Modern textbook language intricacy may confuse the subject, says Pedagogue 2. Pedagogue 3 recommends many materials to meet the challenges university entrance exam. Pedagogue 4 discusses adjusting teaching methods to different learning types, while Pedagogue 5 promotes a diverse instructional toolkit with practical applications to apply theory. Pedagogue 1, struggling students learn logarithms with encouragement and examples. Pedagogy

2 introduces logarithmic expressions using superscript functions, whereas Pedagogy 3 starts with simple examples and progresses to more complex ones. Individual and group interaction is emphasised in Pedagogue 4, and technology and simplified obstacles enhance learning in Pedagogue 5. Teachers also provide some suggestions. Pedagogue 1 states a teacher's enthusiasm for arithmetic may interest students, Pedagogue 2 focuses teachers pre - training, and Pedagogue 3 proposes revisiting logarithmic subjects all year. Pedagogue 4 suggests patience and hard work may engage math-resistant students, whereas Pedagogue 5 focuses making math relevant to real life and gradual, precise teaching to help students with logarithms.

Summary of Pedagogical Perspectives

The findings from mathematics teachers align with the research questions, providing a clear view of the difficulties and challenges students face with logarithms, as well as how these issues manifest in their work and affect their ability to apply mathematical concepts in assignments. Students' difficulties are often tied to a lack of foundational mathematical skills (notably in arithmetic and exponentiation), which becomes particularly evident in their performance on logarithmic tasks. Difficulties also stem from a lack of deep conceptual understanding of logarithms, as students may memorize procedures without truly comprehending the underlying principles.

The pedagogues note that students face challenges in relating logarithmic concepts to real-world scenarios, which is due to the abstract nature of logarithms. Difficulties in transitioning from the memorization of logarithmic rules to their application in varied contexts are common. On other hand, there is a significant variance in student abilities to fluently translate between exponential and logarithmic forms, indicating that while some students quickly grasp these relationships, others struggle significantly.

The challenges lead to a notable decline in correct responses and an increase in non-responses, especially as the difficulty level of the tasks increases. Students' misunderstanding of fundamental properties of logarithms, such as the product rule, impedes their ability to apply these concepts successfully in assignments. The variety of individual aptitudes for mathematics suggests that these challenges can significantly hinder some students from successfully applying logarithmic concepts in assignments, whereas others may overcome them with relative ease.

4.2. Analysis on Performance of Students

The student assessment data was subjected to descriptive statistical analysis to quantify performances by level. The analysis is segmented into three levels of Question difficulty – Easy (Question 1-4), Medium (Question 5-9), and Difficult (Question 10-15). The data elucidates students' performance across these categories, offering insight into their proficiency in logarithmic concepts. The table 1below represent the average percentage of responses by type (True, False, Empty (Not Responded)) and by level (Easy, Medium, Difficult). Firstly, the aim is to determine the optimum True, False and Empty rate values for each category of questions. The maximum (%) of Blank and Incorrect answers and the minimum (%) of Correct answers are important for these analyses and are considered as maximum difficulty questions for further analyses and are highlighted in red in the Table 1 below:

Level	**Question No**	**True (%)**	**False (%)**	**Empty (%)**	***Avg. True by Level (%)***	***Avg. False by Level (%)***	***Avg. Empty by Level (%)***
Easy	*Question 1*	88%	9%	2%	***56%***	***28%***	***15%***
	Question 2	57%	34%	7%			
	Question 3	62%	29%	9%			
	Question 4	**19%**	**38%**	**41%**			
Medium	*Question 5*	69%	22%	9%	***41%***	***23%***	***33%***
	Question 6	**24%**	**33%**	41%			
	Question 7	36%	19%	43%			
	Question 8	29%	16%	**53%**			

	Question 9	48%	28%	21%			
Difficult	*Question 10*	47%	29%	21%	***19%***	***19%***	***60%***
	Question 11	**5%**	16%	**78%**			
	Question 12	**7%**	17%	**74%**			
	Question 13	26%	7%	66%			
	Question 14	**2%**	**40%**	57%			
	Question 15	29%	3%	66%			

Table 2: Performance of Students by Difficulty level of questions and Type of Responses

Source: Calculated by author

As logarithmic problem-solving difficulty grew, student performance fell. At the Easy level, 56% of participants answered the questions correctly, but 28% answered False, indicating fundamental knowledge gaps. Question of medium difficulty had fewer correct answers (41%), more non-responses (33%), indicating an increase in difficulty and possibly avoidance methods. The Difficult level was difficult because accuracy declined to 19% and non-responses rose to 60%. The Easy category shows that most students understand basic ideas, while the Medium and Difficult categories show a deterioration. The need for pedagogical strategies to teach advanced logarithmis is critical. Analysis of student responses to each question by kind followed. The table above shows the percentage of True, False, and Empty replies for each Question by difficulty level.

Table 1 shows numerous patterns. Question 1 in the 'Easy' category was the best for students, with 88% right. Despite this, Question 4 saw a substantial reduction in performance, with just 41% accurate replies, indicating a conceptual gap or difficulty increase. Question 5 had the highest right response rate in the 'Medium' category at 69%, indicating moderate concept understanding. Question 6 had the lowest right response rate (24%), and the highest non-response rate (41%), indicating difficulty or avoidance. The 'Difficult' category was the hardest for students, with all questions below 50% accurate. Note that Question 11 had a 5% right response rate and a 78% non-response rate, while Question 14 had 2% and 40% true and false rates, demonstrating a substantial lack of knowledge or participation. Some questions are left unfilled due to complexity that students have comprehension of the subject. Correctly solved questions or True answers indicate that students had no difficulty with these questions. The questions left blank or Empty responses were obviously "not attempted" to be solved and were therefore left blank. By considering these questions, it would be impossible to determine in which subject the student has difficulty when solving logarithms. On the other hand, it is possible to identify the difficulties experienced by the students by analysing the questions that have been "attempted" to be solved but incorrectly or False responded. The table below shows the percentage of students who gave False reponses, i.e. questions that were "attempted" to be solved by the students. After calculation, ratios are listed descendingly. Considering "attempted" questions with more than 50% false questions can help identify why students struggle to grasp Logarithms. These most challlenging questions were indicated as red color in the Table 1 below:

Question No	**False (% attempted Question)**
Question 1	9%
Question 15	11%
Question 13	21%
Question 5	25%
Question 3	32%
Question 7	34%
Question 8	35%
Question 9	36%
Question 2	38%
Question 10	39%
Question 6	58%
Question 4	67%
Question 12	71%
Question 11	75%
Question 14	96%

Table 3: Proportion of Incorrectly Attempted Logarithm Questions by Students

Source: Calculated by author

As seen in the previous Table 1, questions 4-6-11—12-14 also appear in Table 2 as the questions with the highest degree of incomprehension, unsolved or incorrectly solved for the students. In other words, these questions were incorrectly solved by 67%, 58%, 75%, 71% and 96% of the students respectively. Through the analyses conducted on student performances, it becomes evident that students struggle with comprehending and grasping the concept of logarithm. This is mostly demonstrated by the questions *(Question 4,6,11,12, and 14)* that highlight their difficulties in understanding and learning logarithm. There are some questions

Summary Performance analysis

The percentage of True, False and Emty responses for each questions was calculated through Excel platform using =COUNTIF formula. Then, using =AVERAGE function, the average percentage of True, False and Emty responses for each Level (Easy, Medium, and Difficult) was calculated by dividing the sum of the percentages of the questions for each level by the number of questions corresponding level. The Table 1 shows the results of above calculations. Based on this results for each Level, the lowest percentage of True answers, the highest percentage of False answers, and the highest percentage of Empty answers were taken into account and highlighted in the relevant Tables. The analysis reveals that as the difficulty level of questions increases, the percentage of correct responses ('True') decreases. This suggests students are not fully grasping the logarithmic concepts as they transition to more complex problems. The high percentage of unanswered questions ('Empty') at the Difficult level indicates that students are either overwhelmed by the complexity or lack the confidence to attempt these questions, which reflects in their performance on logarithmic tasks. The analysis indicates that the common challenges students face include a fundamental gap in knowledge, as seen in the Easy level questions, and a potential avoidance of more complex Medium level questions. This could be due to a lack of understanding or a strategic choice to focus on questions where they feel more competent. The challenges influence students' ability to successfully apply logarithmic assignments, as evidenced by the high rate of incorrect

('False') and non-responses. This is particularly noticeable for certain questions where the majority of students who attempted the question answered it incorrectly, indicating a misapplication of logarithmic rules or concepts.

4.3. Error types according Robert Gagne principles

The provided table 3 below categorizes student errors in logarithm questions according to Robert Gagné's (1984) Intellectual Skills taxonomy. The highlighted percentages indicate the type of cognitive process where students are most commonly making errors for each question, divided by difficulty level:

Level	Questions	Discrimination	Concrete Concepts	Rule Using	Problem Solving
EASY	Question 1	0%	0%	100%	0%
	Question 2	0%	100%	0%	0%
	Question 3	0%	0%	0%	100%
	Question 4	17%	0%	83%	0%
Medium	Question 5	0%	100%	0%	0%
	Question 6	0%	0%	100%	0%
	Question 7	0%	100%	0%	0%
	Question 8	0%	100%	0%	0%
	Question 9	0%	100%	0%	0%
Difficult	Question 10	0%	0%	0%	100%
	Question 11	0%	100%	0%	0%
	Question 12	0%	0%	100%	0%
	Question 13	0%	0%	100%	0%
	Question 14	0%	100%	0%	0%
	Question 15	0%	100%	0%	0%

Table 4: Error Types of students based on Gagne (1984)

Source: Calculated by author

At Easy Level Questions (1-4) students predominantly make mistakes in the 'Concrete Concepts' category, with Question 2 having 100% of errors in this category, indicating a lack of understanding of the fundamental concepts of logarithms. Question 3 is an exception, with 100%

of errors in 'Problem Solving', suggesting students understand the concept but struggle with applying it to solve problems. Medium Level Questions (5-9) similar to the Easy level, students again predominantly make errors in the 'Concrete Concepts' category, with all the questions except Question 6 showing 100% of errors here. Question 6 displays 100% of errors in 'Rule Using', which indicates that while students may understand the concepts, they are misapplying the rules or formulas associated with logarithms. Finally at Difficult Level Questions (10-15), there's a more diverse distribution of error types. Questions 10 and 13 show errors solely in 'Problem Solving', meaning students struggle with the application of concepts and rules in complex scenarios. Question 11, 14, and 15 errors fall under 'Concrete Concepts', pointing to fundamental misunderstandings. Question 12 shows errors in 'Rule Using', signifying a difficulty in applying logarithmic rules correctly.

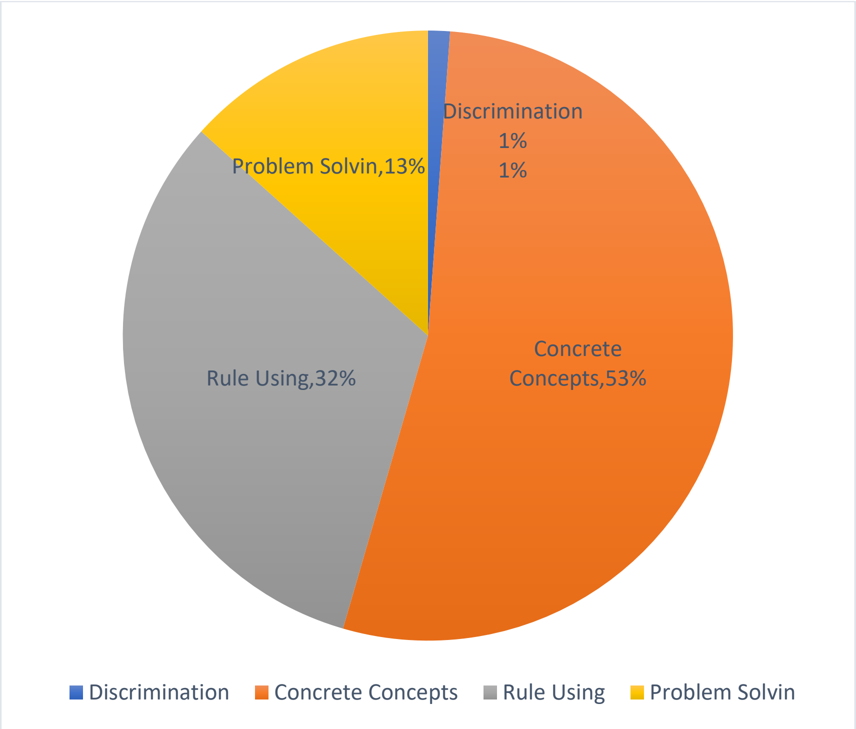

Figure 2: Percantage of Error types according Robert Gagne (1984) Taxanomy

Source: Illustrated by author

The pie chart above in Figure 2represent the average types of student errors in logarithm problems, categorized according to Gagné's taxonomy of learning Concrete Concepts category has the largest proportion of errors, making up 53% of the total. This suggests that more than half of the student errors stem from a fundamental misunderstanding of the basic concepts of logarithms. The second-largest category is the Rule Using, accounting for 32% of errors, indicates that a significant number of students know the concepts but are making mistakes when it comes to applying the rules of logarithms. With 13% of errors, Problem Solving category suggests that a smaller, though still notable, portion of students struggles with applying their knowledge of logarithms to solve problems. Finally, Discrimination is the least common type of error, representing only 1% of the total. It indicates that very few students have difficulty distinguishing between when to use logarithmic operations and when not to.

Summary of Error types analysis

Firstly, the False responses was categorised (4 category vased on Gagne (1984)) according to the mistakes made. Then the percentage of each False responses made in each category was calculated using the formula =COUNTIF. Finally, the averaged percentages of each category was calculated. The error analysis using Gagné's taxonomy provides a clear categorization of the types of cognitive difficulties students encounter, directly addressing the research questions. The analysis shows that the largest share of errors occurs in 'Concrete Concepts', indicating a fundamental misunderstanding of basic logarithmic concepts. This impacts students' performance, as they are unable to correctly apply these concepts in tasks. Difficulties in 'Rule Using' suggest that students are aware of the concepts but make errors in applying the rules, which would directly affect their performance on tasks that require rule application. The 'Problem Solving' errors indicate that some students, even if they understand the rules and concepts, struggle to apply this knowledge to solve complex problems.

Students face challenges primarily with 'Concrete Concepts', struggling to understand the fundamental principles behind logarithms. The challenges in 'Rule Using' reveal that students also struggle with the application of logarithmic rules, even if they theoretically understand the concept.

A smaller yet significant challenge in 'Problem Solving' shows that students can find it difficult to navigate through more complex logarithmic problems that require higher-order thinking.

The challenges in understanding and applying concrete concepts directly impact students' abilities to handle assignments correctly, leading to errors and incomplete responses. Misapplication of rules will cause difficulties in executing assignments accurately, especially in problems where multiple steps and rules are involved. The problem-solving errors indicate that students may not be able to successfully complete assignments that require them to integrate multiple concepts or navigate unfamiliar problem structures.

4.4. Traditional assesment on Extreme Difficulties Students faces

Those that struggle with **question number 4** are mostly ones who lack the necessary understanding to utilize the *product rule* in logarithms or *mixed the rules*. The typical mistake observed is that the students first factorized the numbers inside ***log*** then canceled with the base (index) of logarithm without first showing this factorization as the sum of logarithms. Bu still, they correctly applied the logarithm base property for the second addends of the expression.

$$log_2 48 + log_2 5 \cdot log_5 3 = log_2 2 \cdot 24 + log_2 5 \cdot log_5 3$$

After they canceled, they still went on by repeating the same confused process by factoring as shown below:

$$= log_2 24 + 3 = log_2 2 \cdot 12 + 3 = log_2 12 + 3 = log_2 2^2 \cdot 3 + 3 = 2 \cdot 3 + 3 = 9$$

It’s obvious that the student has a fundamental knowledge of basic logarithmic rules as he can successfully apply the power rule even though he can’t reach the correct result.

The another extreme difficulties that student faced could be understand from mistakes **in the question number 6** $\frac{(log_3 36)^2 - (log_3 4)^2}{log_3 12}$ made by students who couldn't finish or solve the questions.

Students *misused the power rule* when calculating logarithms like $(log_a x)^2$. They needed to remember that $(log_a x)^2$ is not equal to $2log_a x$. On other hand, a few students failed *to apply the logarithmic properties* to simplify logarithmic expressions. For example, they failed to collect the like terms within a logarithm that may be combined or made simpler.

Students also made errors in the simplification process resulted from *misusing logarithmic rules*, such as applying the subtraction rule improperly or failing to recognize when to utilize the power rule. Moreover, they *forgot key logarithmic identities*, such as $log_a a = 1$ which is crucial in simplifying expressions and were not careful enough when dealing with terms inside and outside logarithmic functions. There are also errors by student about to forgot to apply the *change base formula* such as $log_a b = \frac{log_c b}{log_c a}$. Finally, some made simple arithmetic errors, such as miscalculations occurred which affected the final result.

The mistakes **in the question number 11** such that *"If* $log_2 7 = a,\ log_2 3 = b, log_2 11 = c$ *, express* $log_2 \frac{168}{121}$ *in terms of* a, b, c *"* made by students who couldn't finish or solve the questions were about forgetting the division rule of logarithm, misunderstanding logarithmic properties and forgetting power rule for logarithms.

Instead of applying the division rule of logarithms, some of the students simply divided 168 by 121 and found the approximate number then calculated the logarithm from the base 2 and could not reach the correct result. Some students were confused because they believed they had used the division rule by first separating the numbers with the logarithms.

$$log_2 \frac{168}{121} = \frac{log_2 168}{log_2 121}$$

The other error made was that students forgot the power rule of logarithmic functions $log_a b^c = c \cdot log_a b$ resulted in errors when simplifying the logarithmic expression.

The errors made by students who couldn't complete or misdid the **question 12** such that ''$If\, 4a^2 + 9b^2 = 13ab$, $calculate$ $\frac{lg\,(2a+3b)^2 - lg25}{5lgab}$ '' are the common problems that were also occurred in the question 6. However, apart from the repeated mistakes, another challenge faced by the students is their inability to properly factorize the expressions provided in the problem, even though some were well-acquainted with all the properties of logarithms. There are different mistakes observed from the papers of the students **in the question 14.**

$$9^{log_{27}\sqrt[3]{3} + log_{2-\sqrt{3}}(2+\sqrt{3})}$$

Students incorrectly assumed that $log_{27}\sqrt[3]{3}$ is equal to $\frac{1}{3}$ without recognizing the cube root in the logarithm. Mistakes occurred in handling radical terms, especially when dealing with expressions like $2 - \sqrt{3}$ and $2 + \sqrt{3}$. The forgot that these irrational numbers are conjugate of each other that they could simply knowing that rule. They mistakenly used the base 2 for the second logarithmic term $log_{2-\sqrt{3}}(2 + \sqrt{3})$ instead of recognizing the correct base. Also occurred some simple arithmetic errors, such as miscalculating the sum or product of terms, can occur and impact the overall solution. Some expanded the bracket the inside logarithm by mixing the product rule such that they summed up the terms with the same bases as below:

$$log_{2-\sqrt{3}}(2 + \sqrt{3}) = log_{2-\sqrt{3}}2 + \; log_{2-\sqrt{3}}\sqrt{3}$$

Summary of Analysis on Students Difficulties

The detailed error analysis sheds light on the cognitive processes students struggle with and offers a clearer picture of the gaps in their understanding. Students appear to have a foundational knowledge of logarithmic rules but encounter significant challenges in correctly applying this knowledge to solve logarithmic problems. Students showed a lack of understanding of the product rule in logarithms, indicated by incorrect application and factorization processes. This directly ties to the first research question, highlighting that students' key difficulties lie in applying fundamental logarithmic rules and properties, and these difficulties manifest in their inability to reach correct results despite partially correct processes. The analysis pinpoints common challenges such as misuse of the power rule, incorrect simplification of logarithmic expressions, and a failure to collect like terms within a logarithm. These challenges align with the second research question and demonstrate that students often misapply logarithmic rules or forget to use them altogether. Students' errors in simplification and arithmetic, misapplication of subtraction and power rules, and forgetting crucial logarithmic identities such as the base change formula significantly influence their ability to successfully complete logarithmic assignments. These issues directly showing how the challenges lead to incorrect answers and incomplete solutions in logarithmic assignments.

Chapter V

FINDINGS AND FINAL PRODUCT

5.1. Discussion

The results obtained from the examination of pedagogical viewpoints and student performance align with the existing body of literature that highlights the challenges encountered by students in the process of learning logarithms. The literature suggests that students frequently have difficulties when dealing with abstract mathematical ideas. This aligns with the findings based on teachers reflections, who have seen that pupils find logarithms particularly tough because of their abstract nature (Weber, 2002; Even, 1990; Vinner, 1991). This is consistent with the viewpoint of Mirzəzadə M.N. (2023), who highlights the significance of using specific illustrations to facilitate comprehension of abstract mathematical ideas.

In addition, the literature indicates that including real-life examples and applications might enhance the relevance and appeal of concepts for students (Kadijevich, 2000). This corroborates Pedagogue 5's assertion that pupils lack comprehension of the pragmatic implications of mathematics, hence impeding their motivation and acquisition of knowledge. The findings reinforce the literature's focus on the significance of a strong mathematics base (Pedagogue 2), as students who face difficulties with fundamental concepts also tend to encounter challenges with more complex subjects such as logarithms (Whiteside, 2014).

Utilising Gagné's taxonomy for error analysis, it was discovered that most student errors were concentrated in the category of 'Concrete Concepts', suggesting a profound lack of comprehension regarding fundamental logarithmic principles. According to the literature, students often feel frustrated when trying to grasp logarithms, as indicated by Whiteside (2014).

Furthermore, the study reveals that students have a grasp of the concepts but struggle with correctly implementing the rules. This finding supports Mayer's argument for a balanced approach to teach students, which combines abstract theory with real-life examples. The need for instructional solutions to overcome these obstacles is also underlined in the research, which advocates integrating visual representations and technology to promote learning (Nardi, 2000; Tall, 1992).

The congruence of the study's findings with the literature shows that educators should focus on tactics that address the basic deficiencies in students' understanding of logarithms and boost their ability to apply rules correctly. This includes offering specific examples, incorporating technology, and providing individualized help. Future study could evaluate the effectiveness of various instructional strategies, such as inquiry-based learning or flipped classroom models, in addressing students' issues with logarithms (Artigue & Blomhøj, 2013; Prince & Felder, 2006).

Additionally, considering the significant role of affective factors like motivation and anxiety in learning outcomes (Pajares, 1996), future research could investigate strategies to address these factors, such as promoting a growth mindset or incorporating metacognitive strategies into logarithm instruction (Dweck, 2006; Schraw et al., 2006).

5.2. Final Product

A synthesis of teacher views and academic analyses reveals comprehensive teaching strategies aimed at developing students' understanding of logarithms. Initially, a strong emphasis should be placed on building a solid foundation in basic mathematical concepts. That is, logarithms should be introduced in relation to pre-existing knowledge, supported by progressive learning with practical examples and real-life applications, and include contextualised learning that makes the concepts concrete and engaging. The importance of real-world applications to trigger students' curiosity and motivation is emphasised, but it is crucial to foster a supportive and collaborative learning environment. Encouraging group work, discussions, and ongoing assessment and feedback are suggested strategies for identifying and promptly addressing individual student difficulties.

Reflection on student performance and error analysis illustrate the need for a strong conceptual foundation with interactive teaching methods that go beyond rote memorisation. Special focus should be placed on addressing common misconceptions and providing numerous examples of increasing complexity, thus helping students to recognise and avoid common mistakes.

It is also very important to support students' problem-solving skills and critical thinking. Students should methodically analyse problems, identify the necessary logarithm rules and apply them skilfully. Encouraging students to engage in reflective learning and self-correction after problem solving, possibly through peer assessment sessions or guided discussions, is seen as an important means of deepening their understanding of logarithmic concepts.

Graphing calculators, educational software and online platforms can provide visual representations of logarithmic functions and provide opportunities for interactive problem solving. Furthermore, the development of an interactive and collaborative learning environment where group work and discussions are encouraged can expose students to different thought processes and enhance their overall learning experience, making the journey through the complex world of logarithms not only educational but also engaging.

Finally, the focus should be on a clear understanding of the rules and properties of logarithms and educators should take extra time to explain and illustrate each rule, address common misconceptions and use a variety of examples. It is crucial to develop problem solving skills and critical thinking through a structured problem solving approach and reflective learning. It is essential to promote an interactive and collaborative learning environment with the integration of technology and interactive tools to make the abstract nature of logarithms more accessible and the learning journey more dynamic and engaging.

Chapter VI

CONCLUSION

The research aimed to investigate the significant challenges students encounter in understanding and applying logarithms, a concern highlighted by the poor performance in logarithm-related tasks observed in the State Examination Centre (SEC) mathematics exam in the Republic of Azerbaijan. The study's objectives were to identify these challenges, understand their underlying causes, and explore effective teaching methodologies to enhance student learning and performance in logarithms. The study revealed that students face multifaceted difficulties with logarithms. Primarily, the abstract nature of logarithms presents a significant hurdle, with students struggling to comprehend and apply the fundamental principles in practical contexts. The research highlighted a substantial gap between students' theoretical understanding and their practical application skills. This gap was evidenced by numerous errors categorized under 'Concrete Concepts', indicating a deep-seated misunderstanding of the core principles of logarithms.

In exploring the reasons behind these difficulties, the research pinpointed a weak mathematical foundation, particularly in basic arithmetic and algebra, as a primary obstacle. The conventional teaching methods, which may not effectively address the abstract and intricate nature of logarithms, were also noted as a contributing factor. Cognitive challenges in applying theoretical knowledge to practical problems were evident, suggesting issues in transitioning from understanding to application. Moreover, the study recognized the influence of individual variability in innate mathematical abilities, learning styles, and prior knowledge on the students' grasp of logarithms.

The study also delved into various teaching methodologies and strategies that educators employ to facilitate students' understanding of logarithms. Emphasizing real-world examples and practical applications was seen as critical in making the concept of logarithms more accessible and comprehensible. The use of visual aids and technology-enhanced tools, including graphing calculators and educational software, was highlighted for their effectiveness in supporting teaching and learning. The importance of differentiated instruction was acknowledged for addressing the diverse learning needs and paces of students. Furthermore, continuous assessment was recommended as a means to monitor student understanding and provide timely feedback. The study stressed the need for a conceptual focus, advocating for teaching approaches that prioritize deep understanding over rote memorization. Additionally, the necessity for professional development opportunities for teachers was underscored, emphasizing the need for educators to be equipped with the latest and most effective teaching strategies.

REFERENCES

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. ZDM, 45(6), 797-810.

Babbie, E. (2016). The practice of social research (14th ed.). Boston, MA: Cengage Learning.

Black, P., & Wiliam, D. (1998). Assessment and classroom learning. Assessment in Education: Principles, Policy & Practice, 5(1), 7-74.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. Qualitative Research in Psychology, 3(2), 77-101.

Briggs, H. (1624). Arithmetica Logarithmica. London.

Byerley, C., Hatfield, N. J., & Thompson, P. W. (2015). Calculus students' understandings of division and rate. In T. Fukawa-Connelly, N. E. Infante, K. Keene, & M. Zandieh (Eds.), Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education (pp. 104-109). RUME.

Confrey, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. Educational Studies in Mathematics, 26(2-3), 135-164.

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). Introduction to algorithms. MIT press.

De Bock, D., Verschaffel, L., & Janssens, D. (2000). The predominance of the linear model in secondary school students' solutions of word problems involving length and area of similar plane figures. Educational Studies in Mathematics, 43(1), 61-83.

Dreyfus, T., & Eisenberg, T. (1986). On the aesthetics of mathematical thought. For the Learning of Mathematics, 6(1), 2-10.

Dubinsky, E., Elterman, F., & Gong, C. (2005). The student's construction of quantification. For the Learning of Mathematics, 25(2), 15-23.

Dubinsky, E., Weller, K., McDonald, M. A., & Brown, A. (2005). Some historical issues and paradoxes regarding the concept of infinity: An Apos-based analysis: Part 1. Educational Studies in Mathematics, 58(3), 335-359.

Dweck, C. S. (2006). Mindset: The new psychology of success. Random House.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational studies in mathematics*, *21*(6), 521-544..

Feynman, R. P., Leighton, R. B., & Sands, M. (2011). The Feynman lectures on physics. Basic Books.

Field, A. (2018). Discovering statistics using IBM SPSS statistics (5th ed.). London, UK: SAGE Publications Ltd.

Hohenwarter, M., Hohenwarter, J., & Lavicza, Z. (2008). Introducing dynamic mathematics software to secondary school teachers: The case of GeoGebra. Journal of Computers in Mathematics and Science Teaching, 27(2), 135-152.

Kadijevich, D. M. (2000). Connecting mathematics with work and life. Washington, DC: National Council of Teachers of Mathematics.

Katz, V. J. (2009). A history of mathematics: An introduction. Pearson.

Kellison, S. G. (2009). The theory of interest. McGraw-Hill/Irwin.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. Review of Educational Research, 60(1), 1-64.

Napier, J. (1614). Mirifici Logarithmorum Canonis Descriptio. Edinburgh.

Nardi, E. (2000). Mathematics undergraduates' responses to semantic abbreviations, geometric images and multi-level abstractions in Group Theory. Educational Studies in Mathematics, 43(2), 169-189.

Nicol, D. J., & Macfarlane-Dick, D. (2006). Formative assessment and self-regulated learning: A model and seven principles of good feedback practice. Studies in Higher Education, 31(2), 199-218.

Pajares, F. (1996). Self-efficacy beliefs in academic settings. Review of Educational Research, 66(4), 543-578.

Prince, M. J., & Felder, R. M. (2006). Inductive teaching and learning methods: Definitions, comparisons, and research bases. Journal of Engineering Education, 95(2), 123-138.

Schraw, G., Crippen, K. J., & Hartley, K. (2006). Promoting self-regulation in science education: Metacognition as part of a broader perspective on learning. Research in Science Education, 36(1-2), 111-139.

Stewart, J. (2016). Calculus(8th ed.). Cengage Learning.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity, and proof. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 495-511). Macmillan.

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. Tall (Ed.), Advanced mathematical thinking (pp. 65-81). Dordrecht: Kluwer Academic Publishers.

Weber, K. (2002). Beyond the cognitive: Using a system perspective to explain the formation and dissolution of mathematical conceptions. Journal for Research in Mathematics Education, 33(2), 137-157.

Weber, K. (2002). Students' difficulties with proof. School Science and Mathematics, 102(7), 343-356.

Creswell, J. W., & Creswell, J. D. (2017). Research design: Qualitative, quantitative, and mixed methods approaches. SAGE Publications.

Abbaszadə, M. M., Bədəlov, T. A., & Şelaginov, O. Y. (2020). 2018/2019-cu TƏDRİS İLİ ÜÇÜN AZƏRBAYCAN RESPUBLİKASININ ALİ VƏ ORTA İXTİSAS TƏHSİLİ MÜƏSSİSƏLƏRİNƏ TƏLƏBƏ QƏBULU VƏ ÜMUMTƏHSİL MÜƏSSİSƏLƏRİ ÜZRƏ BURAXILIŞ İMTAHANLARI NƏTİCƏLƏRİNİN ELMİ-STATİSTİK TƏHLİLİ *[Scientific-statistical analysis of the results of "Student admission to higher and secondary educational institutions of the Republic of Azerbaijan and graduation exams for general education institutions"* *for the 2018/2019 academic year.].* Abiturient, (12), 2018. Retrieved from State Exam Centre (SEC) of Azerbaijan, 2020. https://dim.gov.az/activities/research/statistika/

Cəlilova V.M. (2021). "RİYAZİ ANALİZİN ÖYRƏDİLMƏSİNDƏ İNFORMASİYA TEXNOLOGİYALARINDAN İSTİFADƏ METODİKASI" [METHODOLOGY OF USING INFORMATION TECHNOLOGIES IN TEACHING MATHEMATICAL ANALYSIS], Magistrantların XXI Respublika Elmi konfransı, 17-18 may 2021-ci il. [XXI Republican Scientific Conference of Masters, May 17-18, 2021.]

Mirzəzadə M.N. (2021). "RİYAZİ ANALİZ ANLAYIŞLARININ FORMALAŞMASININ BƏZİ MƏSƏLƏLƏRİ" [Some Problems of the Formalization of Mathematical Analysis Concepts], Magistrantların XXI Respublika Elmi konfransı, 17-18 may 2021-ci il. [XXI Republican Scientific Conference of Masters, May 17-18, 2021.]

Fərmanlı Ü.İ. (2021). YENİ TƏLİM SİSTEMİNDƏ PROBLEMİN QOYULUŞU VƏ MƏKTƏB TƏCRÜBƏSİNDƏ VƏZİYYƏT" [STATEMENT OF THE PROBLEM IN THE NEW EDUCATIONAL SYSTEM AND SITUATION IN SCHOOL EXPERIENCE], Magistrantların XXI Respublika Elmi konfransı, 17-18 may 2021-ci il. [XXI Republican Scientific Conference of Masters, May 17-18, 2021.]

Səlimli Z.V. (2021). "RİYAZİ METODLAR ELMLƏRİN RİYAZİLƏŞMƏSİ VASİTƏSİ KİMİ'', [Mathematical Methods as a Means of Mathematization of Sciences] , Magistrantların XXI Respublika Elmi konfransı, 17-18 may 2021-ci il. [XXI Republican Scientific Conference of Masters, May 17-18, 2021.]

Gagne, R. M. (1984). Learning outcomes and their effects: Useful categories of human performance. American psychologist, 39(4), 377. http://davidlewisphd.com/courses/8001/EDD8001/SUM12/1984-Gagne.pdf

Gagné, R.M., & Briggs, L.J. (1974). Principles of instructional design (2nd ed.). Holt, Rinehart, and Winston.

Reigeluth, C. M. (1983). Instructional design theories and models: An overview of their current status. Hillsdale, NJ: Lawrence Erlbaum Associates.

Seels, B., & Glasgow, Z. (1990). Exercises in instructional design. Columbus, OH: Merrill Publishing Company.

Mayer, R. E. (2004). Should there be a three-strikes rule against pure discovery learning?. American psychologist, 59(1), 14. https://www.sfu.ca/~jcnesbit/EDUC220/ThinkPaper/Mayer2004.pdf

Boaler, J. (2015). Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching. John Wiley & Sons. https://www.creatingrounds.com/uploads/9/6/2/4/96240662/mathematical_mindsets.pdf

Van de Walle, J. A. (2013). Elementary and middle school mathematics: teaching developmentally/John A. Van de Walle, Karen S. Karp.

Mayer, R. E. (2004). Should there be a three-strikes rule against pure discovery learning?. American psychologist, 59(1), 14. https://www.sfu.ca/~jcnesbit/EDUC220/ThinkPaper/Mayer2004.pdf

Boaler, J. (2002). Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning. Routledge.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. Second handbook of research on mathematics teaching and learning, 1(1), 371-404. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=3b2e2aabd07c64bb65408a3891902be4b7277cd6

Drijvers, P., Kieran, C., Mariotti, M. A., Ainley, J., Andresen, M., Chan, Y. C., ... & Meagher, M. (2010). Integrating technology into mathematics education: Theoretical perspectives. Mathematics education and technology-rethinking the terrain: The 17th ICMI Study, 89-132. https://d1wqtxts1xzle7.cloudfront.net/42054279/Integrating_Technology_into_Mathematics_20160204-20783-1e11uxa-libre.pdf?1454601770=&response-content-

disposition=inline%3B+filename%3DIntegrating_Technology_into_Mathematics.pdf&Expires=1697455342&Signature=HI76udfhK-6kQL14awvHH-4eYyIihSusKr1n7~99hZ~R5Jt0Co8tekd80D8BlTuSWlxgI1BeAF-qlMBkEQDMfH0T1A~OzH8sdC01v7VGE05CsuV87XZGcVHpYh~v-v6OJSgAusGdZTnhVTbNxH5C4GIZnzd0QWDBcCwfn4LiepbLFdV-IL3pOW4LDok0e8xgUvmv4Gf7vBjPptbmkz1n5YP~3GiNgwPKJwJbTqi5E0imiERwhbE1ZsKJRUnLMdjL9dDp60kQHNnnMo0VLm~2R5WgsMpIht7Z9~9~RuCi9t1NAJo-GIMRWfKSRsbOihlFR7avVoi2RueQ58sN0Kzhlw__&Key-Pair-Id=APKAJLOHF5GGSLRBV4ZA

Dintarini, M. (2018). Understanding Logarithm: What are the Difficulties That Students Have?. https://www.atlantis-press.com/article/25901746.pdf

Farkas, G. (2003). Cognitive skills and noncognitive traits and behaviors in stratification processes. Annual review of sociology, 29(1), 541-562. https://www.annualreviews.org/doi/abs/10.1146/annurev.soc.29.010202.100023

Ferrini-Mundy, J. (2000). Principles and standards for school mathematics: A guide for mathematicians. Notices of the American Mathematical Society, 47(8). https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=7f3b042c8337fe9195ed6ca0cb017b76bbf1ff7c

Chua, B. L., & Wood, E. (2005). Working with logarithms: Students' misconceptions and errors. https://repository.nie.edu.sg/server/api/core/bitstreams/8efe2924-628c-4058-9460-e2e4d8fde45f/content

Hirsch, J., & Pfeil, J. (2012). On Teaching Logarithms using the Socratic Pedagogy. MATHEMATICS TEACHING-RESEARCH JOURNAL ONLINE, 5(4). https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=6ea6d3726807070c73a156584f4924aed67734f7

Jenifer, J. B., Levine, S. C., & Beilock, S. L. (2023). Studying while anxious: mathematics anxiety and the avoidance of solving practice problems during exam preparation in college calculus. ZDM–Mathematics Education, 55(2), 359-369. https://link.springer.com/article/10.1007/s11858-022-01456-1

Ansah, I. (2016). Improving understanding of logarithms by using the approach of repeated division (Master's thesis, Universitetet i Agder; University of Agder). https://uia.brage.unit.no/uia-xmlui/bitstream/handle/11250/2412206/Ansah,%20Isaac.pdf?sequence=1

Seery, C., Andres, A., Moore-Cherry, N., & O'Sullivan, S. (2021). Students as partners in peer mentoring: Expectations, experiences and emotions. Innovative Higher Education, 46(6), 663-681. https://link.springer.com/article/10.1007/s10755-021-09556-8

Radović, S., Marić, M., & Passey, D. (2019). Technology enhancing mathematics learning behaviours: Shifting learning goals from "producing the right answer" to "understanding how to address current and future mathematical challenges". Education and Information Technologies, 24, 103-126. https://link.springer.com/article/10.1007/s10639-018-9763-x

Unlu, V. (2017). How to create an inclusive classroom environment. World of Better Learning. https://www.cambridge.org/elt/blog/2017/11/15/create-inclusive-classroom-environment/

Whiteside, D. T. (2014). And John Napier created logarithms... BSHM Bulletin: Journal of the British Society for the History of Mathematics, 29(3), 154-166.

Lakens, D. (2022). Sample size justification. Collabra: Psychology, 8(1), 33267. https://online.ucpress.edu/collabra/article/8/1/33267/120491/Sample-Size-Justification Zamin Oruczade [OZAN] YouTube Channel (March 22, 2022). buraxılış imtahanlarında istifadə olunan üstlü və loqarifmə aid suallar və izahı: https://youtu.be/KctOqERN5-Q?si=ZzOAxES1Ab1q0qP4

Gunay Ceferova [Riyaziyyat dersleri] YouTube Channel (March 5, 2023) 5 mart 2023 Buraxılış imtahanı 11-ci sinif Riyaziyyat suallarının izahı (1-25)05.03.2023: https://youtu.be/fKpoV0XU97s?si=jGZABdvbFVBd7ODp

Gunay Ceferova [Riyaziyyat dersleri] YouTube Channel (March 12, 2023) 12 mart 2023 Buraxılış imtahanı 11-ci sinif Riyaziyyat suallarının izahı (1-25)12.03.2023: https://youtu.be/rCnzn5X_Oeg?si=HF5gpfJPPZoSVgxq

APPENDIXES

Appendix A. Semi-structured interview questions for teachers

General Introduction & Purpose of Study

The interview that will be taken from you will justify the level at which the students have mastered the subject of logarithm in mathematics and the general difficulties they face in this subject based on your experience and the recommendations you will give to overcome these difficulties. The purpose of this study is to reveal the factors that cause the difficulties faced by 10th grade students in the subject of logarithms and to investigate how these difficulties can be solved with the support of schoolteachers.

Mathematics Teacher Interview Consent Form

Interviewee Information
Name:
School:
Mathematics Teaching Experience:

Interview Questions & Responses

Could you give a brief information about yourself, please? How long have you been working as a math teacher?

How do you determine if your students have math difficulties?

Do teaching materials and methodology help students master the subject?

Do you think your students do not master the subject of logarithms well?

In general, what are the common problems they face in understanding the concept of logarithms?

In your opinion, what are the main reasons for these difficulties?

What methodology do you use to help students learn logarithm better?

Based on your experiences, can students easily see the relationship between exponential and logarithmic equations such as $a^y = x$ və $log_a x = y$ and build them fluently? More specifically, do they know that $a^y = x$ və $log_a x = y$ are equivalent notations?

Can they master and apply the fundamental properties of logarithms such as $log_c xy = log_c x + log_c y$ well?

What recommendations do you have for teachers to support students as they struggle to master logarithms?

Appendix B. Interview Consent Form

Interview Question	Respondent	Response Summary	Codes
Brief information about yourself?	Teacher 1	15 years experience, focuses on algebra and calculus	[Teacher Experience] [Subject Specialization]
	Teacher 2	8 years experience, specializes in geometry and trigonometry	[Teacher Experience] [Subject Specialization]
	Teacher 3	20 years experience, comprehensive math teaching	[Teacher Experience] [Broad Subject Knowledge]
	Teacher 4	5 years experience, recently started teaching logarithms	[Teacher Experience] [New to Topic]
	Teacher 5	12 years experience, emphasizes practical applications	[Teacher Experience] [Practical Approach]
How do you determine students' math difficulties?	Teacher 1	Observes performance in tests and assignments	[Assessment-Based Observation]
	Teacher 2	Notes class participation and student questions	[Class Participation] [Student Queries]
	Teacher 3	Uses quizzes and one-on-one meetings	[Frequent Quizzes] [Individual Assessment]
	Teacher 4	Relies on student feedback and group activities	[Student Feedback] [Group Work]
	Teacher 5	Monitors homework and classwork consistency	[Homework Review] [Classwork Consistency]
Do materials and methodology help in mastering math?	Teacher 1	Finds materials adequate but not sufficiently engaging	[Material Adequacy] [Lack of Engagement]
	Teacher 2	Believes materials are outdated, prefers interactive methods	[Outdated Materials] [Preference for Interactivity]
	Teacher 3	Satisfied with materials, but suggests more real-life examples	[Material Satisfaction] [Need for Real-life Examples]
	Teacher 4	Feels current methodology is too theoretical	[Theoretical Approach]
	Teacher 5	Uses a mix of textbooks and online resources for better understanding	[Mixed Resources] [Online Integration]
Do students master logarithms well?	Teacher 1	Not particularly, students find logarithms abstract	[Abstract Nature]
	Teacher 2	Some do, but many struggle with application	[Mixed Understanding] [Application Struggle]
	Teacher 3	Majority find it challenging, especially higher-order problems	[General Difficulty] [Complex Problem Struggles]
	Teacher 4	Observes improvement with hands-on activities	[Improvement with Practical Work]
	Teacher 5	Notes that continuous practice leads to better mastery	[Importance of Practice]
Common problems in understanding logarithms?	Teacher 1	Confusion between logarithmic and exponential forms	[Form Confusion]

	Teacher 2	Difficulty applying logarithm rules in complex problems	[Rule Application Difficulty]
	Teacher 3	Lack of foundational understanding of logarithms	[Foundational Misunderstanding]
	Teacher 4	Struggles with interpreting logarithmic graphs	[Graph Interpretation Difficulty]
	Teacher 5	Issues with transitioning from theory to practice	[Theory-Practice Gap]
Main reasons for difficulties?	Teacher 1	Abstract nature and lack of context in examples	[Abstract Nature] [Lack of Context]
	Teacher 2	Insufficient practice and rote learning	[Insufficient Practice] [Rote Learning]
	Teacher 3	Overemphasis on memorization rather than understanding	[Overemphasis on Memorization]
	Teacher 4	Limited exposure to varied problem types	[Limited Problem Variety]
	Teacher 5	Gap in linking logarithms to real-life applications	[Real-life Application Gap]
Methodology to help learn logarithm better?	Teacher 1	Integrates technology and interactive activities	[Technology Integration] [Interactive Activities]
	Teacher 2	Focuses on problem-solving and critical thinking	[Problem-Solving Focus] [Critical Thinking Emphasis]
	Teacher 3	Uses a step-by-step approach to teach concepts	[Step-by-Step Approach]
	Teacher 4	Encourages group discussions and peer learning	[Group Discussions] [Peer Learning]
	Teacher 5	Regular quizzes and feedback sessions	[Regular Quizzes] [Feedback Sessions]
Relationship between exponential and logarithmic equations?	Teacher 1	Mixed understanding among students	[Mixed Understanding]
	Teacher 2	Most students understand after targeted lessons	[Understanding Post-Instruction]
	Teacher 3	Students struggle to see the connection	[Connection Struggle]
	Teacher 4	Visual aids help in understanding the relationship	[Visual Aids Effectiveness]
	Teacher 5	Continuous practice needed for clarity	[Need for Continuous Practice]
Mastery of logarithm properties?	Teacher 1	Students are generally good with basic properties	[Good with Basics]
	Teacher 2	Mistakes common in applying properties to complex problems	[Mistakes in Complex Applications]
	Teacher 3	Students need more practice with properties	[Need More Practice]
	Teacher 4	Varies greatly among students	[Varied Mastery]
	Teacher 5	Consistent practice leads to better mastery	[Improved with Practice]
Recommendations for teachers?	Teacher 1	Suggests more real-world examples	[Real-world Examples]
	Teacher 2	Recommends interactive and engaging methods	[Interactive Methods]
	Teacher 3	Emphasizes the need for foundational understanding	[Focus on Foundations]
	Teacher 4	Advises on using varied teaching aids	[Varied Teaching Aids]
	Teacher 5	Stresses on continuous assessment and feedback	[Continuous Assessment] [Feedback Importance]

Appendix C. Logarithm assessment questions for students

Easy Level:

Question 1 $log_4 9 + log_2 \frac{4}{3}$ calculate. (2020)

A) 2 B) 4 C) 3 D) 5 E) 1

Question 2 $5^{\frac{2}{log_7 5}}$ calculate. (2019)

A) 25 B) 14 C) 10 D) 7 E) 49

Question 3 $log_6(log_4(log_2 x)) = 0$ (2019)

A) 8 B) 32 C) 64 D) 16 E) 4

Question 4 $log_2 48 + log_2 5 log_5 3$ calculate the value. (2020)

Medium Level:

Question 5 $\sqrt[3]{25}^{log_5 27} - \sqrt[3]{\sqrt{3}}^{log_3 64}$ calculate the value. (2021)

A) 9 B) 5 C) 7 D) 2 E) 3

Question 6 $\frac{(log_3 36)^2 - (log_3 4)^2}{log_3 12}$ calculate the value. (2020)

Question 7 $log_{\sqrt{5}} 3(log_2 32)$ calculate the value. (2022)

Question 8 $\frac{2log_c a + 5 \cdot log_c^{-1} a}{log_c^{-1} a} - 2log_c^2 a$ calculate the value. (2019)

Difficult Level:

Question 9 $log_3(12 - 3^x) = e^{\ln(3-x)}$ solve the equation. (2022)

A) -2 B) 0 C) 2 D) -3 E) 3

Question 10 $log_7(x+0{,}5) = \frac{1}{log_{\frac{9}{x^2}} 49}$ solve the equation.(2021)

A) $-2; 1{,}5$ B) -2 C) $1{,}5$ D) 3 E) $-1{,}5$

Question 11 If $log_2 7 = a,\ log_2 3 = b, log_2 11 = c$,express the $log_2 \frac{168}{121}$ with a, b, c . (2023)

Question 12 Əgər $4a^2 + 9b^2 = 13ab$ olarsa, $\frac{\lg (2a+3b)^2 - lg25}{5lgab}$ calculate the value. (2022)

Question 13 $log_2 x + log_2(2+x) = 2\, log_2\sqrt{3}$ solve the equation. (2019)

Question 14 $9^{log_{27}\sqrt[3]{3}} + log_{2-\sqrt{3}}(2+\sqrt{3})$ calculate the value. (2020)

Question 15 $log_3(36 - 3^x) = 3^{log_3 2}$ solve the equation. (2020)